

Kerr-Schild photonlike metric solutions

Alexander Burinskii

Gravity Research Group, NSI Russian Academy of Sciences,
B. Tulskaya 52 Moscow 115191 Russia;

The charged and spinning lightlike solutions are obtained and analyzed in the Kerr-Schild formalism. One of them may be considered as ultrarelativistic boost of the Kerr-Newman solution along the direction of angular momentum. The Kerr singular ring disappears, going to infinity. However, there remains a finite relativistic parameter a which determines the twist of Kerr congruence and total spin of the solutions by the Kerr relation $J = ma$. In particular, assuming that this solution describes gravitational field of a photon and setting $J = \hbar$ and $E = m$, one obtains that a is de Broglie wavelength. Electromagnetic field of the solutions is aligned with the Kerr null congruence. Some of the presented solutions contain singular lightlike beams.

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I. INTRODUCTION

The problem of finding the ultrarelativistic limits of exact particle-like or BH- solutions of the Einstein field equations have paid attention in the connection with some non-trivial gravitational effects which are expected to occur in the interparticle interactions at extreme energies due to the presence of gravitational shock waves [1, 2] and singular strings. Another field of application is the study of possible influence of gravitational field of photon.

First results in this directions were obtained by Aichelburg and Sexl [3], who considered the behavior of the Schwarzschild metric under ultrarelativistic boost. The resulting metric is a pp-wave, [1, 5],

$$ds^2 = dudv + f(x, y)\delta(v)dv^2 + dx^2 + dy^2 \quad (1)$$

with a front surface $v = 0$ moving along the z axis. Here $v = (z - t)2^{-1/2}$ and $u = (z + t)2^{-1/2}$. In particular, this solution may also be considered as a metric of the Kerr-Schild class

$$g_{\mu\nu} = \eta_{\mu\nu} + 2H(v, x, y)k_\mu k_\nu, \quad (2)$$

where $k^\mu = dv = (dz - dt)2^{-1/2}$ is the expansion-free and twist-free principal null congruence, and $\eta_{\mu\nu}$ is metric of an auxiliary Minkowski space-time. Similar treatments with ultrarelativistic boost of the Kerr geometry have also been performed in many other works [6, 7, 8], leading to the similar conclusion that limiting metrics have the expansion-free and twist-free pp-waves with an energy density $T_{vv} = \rho(x, y)\delta(v)$ distributed on singular front. If the total mass-energy m distributed on the front is finite, then the rest-mass m_0 of the corresponding stationary solution has to tend to zero to provide the finite value for the relativistic mass-energy $E = m = m_0/\sqrt{1 - (v/c)^2}$ in the limit $v \rightarrow c$. Therefore, the limit $v \rightarrow c$ has to be related with the simultaneous limit $m_0 \rightarrow 0$. [27]

If angular momentum of the Kerr solution is oriented along the boost direction, the Kerr singular ring will be

orthogonal to the boost and will not be subjected to Lorentz contraction. During the limiting procedure its radius will be $a = a_0$. Assuming that the rest mass of the prototype Kerr solution is infinitesimally small, i.e. $m_0 \rightarrow 0$ by $v \rightarrow c$, and following the usual Kerr metric relation $J = m_0 a_0$, we obtain that the limiting ultrarelativistic metric will have zero angular momentum, $\lim J|_{m_0 \rightarrow 0} = 0$. We arrive at the conclusion that the most of the obtained ultrarelativistic analogs of the Kerr solution have the zero angular momentum.

However, one can also perform an alternative ultrarelativistic limit of the Kerr solution, setting $m_0 \rightarrow 0$, by the constant value of the total angular momentum $J = m_0 a_0$. Obviously, such a limit has to be accompanied by simultaneous limit $a_0 \rightarrow \infty$, where a_0 is the radius of singular ring of the corresponding Kerr solution at rest. In this case, one defines a ‘relativistic’ parameter $a = J/m$ and retains during the limit the Kerr relation

$$J = m_0 a_0 = ma. \quad (3)$$

One sees that it may be achieved setting $a = a_0 \sqrt{1 - (v/c)^2}$. Such a limit for the uncharged Kerr solution was performed in [9, 10] by using the complex Kerr formalism, and corresponding exact solution may also be obtained in the standard Kerr-Schild formalism. This solution has a finite length parameter a , however, the Kerr singular ring disappears, going to infinity in accordance with

$$a_0 = a/\sqrt{1 - (v/c)^2}. \quad (4)$$

Recently, interest to the light-like solutions with angular momentum was renewed, and there appears series of the works on the ‘gyraton’ solutions, based on the Brinkman class of metrics [11] $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, which is relative to the Kerr-Schild one, but has another type of the quasi-linear contribution $h_{\mu\nu}$.

In this paper we generalize the obtained earlier in [9, 10] solutions, incorporating electromagnetic field which is aligned with respect to Kerr congruence. Note, that in the gyraton solutions electromagnetic field is aligned with

respect to the Killing vector. In conclusion we discuss some new problems which may be related to application of these solutions. In particular, the problem of the obtaining more general solutions with wave electromagnetic field which are related to case $\gamma \neq 0$ of the Kerr-Schild formalism, and also based on the Kerr theorem twistorial approach to interaction between the lightlike and massive particles [12].

II. KERR-SCHILD FORMALISM.

We follow to formalism and notations of seminal work [13]. Metric has the Kerr-Schild form

$$g_{\mu\nu} = \eta_{\mu\nu} + 2he_{\mu}^3e_{\nu}^3, \quad (5)$$

where $\eta_{\mu\nu} = \text{diag}\{-1, 1, 1, 1\}$ is the metric of auxiliary Minkowski background $x^{\mu} = (t, x, y, z) \in M^4$.

The vector field e_{μ}^3 is tangent to principal null congruence which plays central role in the Kerr-Schild formalism, defining the structure of all tensor quantities. It is described in the null Cartesian coordinates

$$\begin{aligned} 2^{\frac{1}{2}}\zeta &= x + iy, & 2^{\frac{1}{2}}\bar{\zeta} &= x - iy, \\ 2^{\frac{1}{2}}u &= z + t, & 2^{\frac{1}{2}}v &= z - t. \end{aligned} \quad (6)$$

and has the form

$$e^3 = du + \bar{Y}d\zeta + Yd\bar{\zeta} - Y\bar{Y}dv. \quad (7)$$

The principal null congruence is determined by the *Kerr theorem* in terms of the complex function $Y \equiv Y(x^{\mu})$, $x^{\mu} \in M^4$. Besides the congruence, function $Y(x^{\mu})$ determines a null tetrad. Direction e^3 is completed to null tetrad e^a , $a = 1, 2, 3, 4$,

$$e^1 = d\zeta - Ydv, \quad e^2 = d\bar{\zeta} - \bar{Y}dv, \quad e^4 = dv + he^3. \quad (8)$$

The null congruences (7) satisfying the conditions

$$Y_{,2} = Y_{,4}, \quad (9)$$

are geodesic and shear-free. (Here $,_a = e_a^{\mu}\partial_{\mu}$ are the directional derivatives [28]).

The Kerr theorem claims that such congruences are determined by function $Y = Y(x^{\mu})$ which is a solution of the equation

$$F(Y, l_1, l_2) = 0, \quad (10)$$

where F is an arbitrary analytic function of the projective twistor coordinates

$$Y, \quad l_1 = \zeta - Yv, \quad l_2 = u + Y\bar{\zeta}. \quad (11)$$

Integration of the Einstein-Maxwell field equations for the ansatz (5) with a general geodesic and shear-free null

field $e_{\mu}^3(x)$, $x \in M^4$ was fulfilled in [13], leading to the following form of the function h :

$$h = \frac{1}{2}M(Z + \bar{Z}) - \frac{1}{2}A\bar{A}Z\bar{Z}. \quad (12)$$

Therefore, the function M, A, \bar{A}, Z and \bar{Z} determine fully the metric, while the strength tensor of self-dual electromagnetic field $\mathcal{F}_{\mu\nu} = \mathcal{F}_{ab}e^a e^b$ is determined by the functions A and γ and has the following nonzero tetrad components

$$\mathcal{F}_{12} = \mathcal{F}_{34} = AZ^2, \quad \mathcal{F}_{31} = \gamma Z - (AZ)_{,1}. \quad (13)$$

The function Z^{-1} characterizes a radial distance which is determined by the Kerr theorem and will be discussed later. Differential equations for the unknown so far functions contain **electromagnetic sector**:

$$A_{,2} - 2Z^{-1}\bar{Z}Y_{,3}A = 0, \quad A_{,4} = 0, \quad (14)$$

$$\mathcal{D}A + \bar{Z}^{-1}\gamma_{,2} - Z^{-1}Y_{,3}\gamma = 0, \quad \gamma_{,4} = 0. \quad (15)$$

Here

$$\mathcal{D} = \partial_3 - Z^{-1}Y_{,3}\partial_1 - \bar{Z}^{-1}\bar{Y}_{,3}\partial_2. \quad (16)$$

Gravitational sector contains two equations:

$$M_{,2} - 3Z^{-1}\bar{Z}Y_{,3}M = A\bar{\gamma}\bar{Z}, \quad (17)$$

$$\mathcal{D}M = \frac{1}{2}\gamma\bar{\gamma}, \quad M_{,4} = 0. \quad (18)$$

The necessary functions Z, Y , their directional derivatives $,_a$ and parameters are determined by the generating function F of the Kerr theorem. In particular, the complex dilatation

$$Z = P/\tilde{r} \quad (19)$$

is related to ‘complex radial distance’

$$\tilde{r} = -dF/dY. \quad (20)$$

Function $F(Y)$ used in [13] has the general form

$$F \equiv \phi(Y) + (qY + c)l_1 - (pY + \bar{q})l_2 \quad (21)$$

where $\phi = a_0 + a_1Y + a_2Y^2$.

The method developed in the papers [9, 10] allows one to fix explicit values of these parameters corresponding to concrete values of the boost and orientation of angular momentum. The solutions of the main equation (10) can

be found in explicit form and correspond to the Kerr solution up to the Lorentz boost, rotation, and shift of the origin.

Coefficients a_0 , a_1 , a_2 define orientation of angular momentum and the constants p, q, \bar{q}, c are related to Killing vector of the solution. They determine function $P = pY\bar{Y} + qY + \bar{q}\bar{Y} + c$.

In particular, for the Kerr-Newman solution at rest $p = c = 2^{-1/2}$, $q = \bar{q} = 0$ and $P = 2^{-1/2}(1 + Y\bar{Y})$. Spin J is oriented along z axis for $a_0 = a_2 = 0$, and $a_1 = -ia$.

III. CLASS OF THE LIGHT-LIKE SOLUTIONS.

We consider the simplest case, taking the above parameters of the Kerr solution at rest as a prototype of the boosted solution. We have $\phi = -iaY$ which corresponds to orientation of the angular momentum in the z -direction, and we direct collinearly the light-like boost which will be described by the parameters

$$p = q = \bar{q} = 0, \quad c = 1, \quad (22)$$

corresponding to the null Killing direction $\hat{K} = \partial_u$. It leads to solutions which depend explicitly on the light-like coordinate $v = 2^{-1/2}(z - t)$. The functions F and P of the Kerr theorem take in this case the simple form

$$F = -iaY + (\zeta - Yv), \quad P = 1. \quad (23)$$

Solution of the equation $F = 0$ yields

$$Y = \zeta/(ia + v), \quad (24)$$

and function $Y(x)$ determines the principal null congruence e^3 by the relation (20). One sees that congruence has a non-trivial coordinate dependence and has a non-zero expansion θ and twist ω determined by

$$Z = \theta + i\omega = (v - ia)/[v^2 + a^2], \quad (25)$$

which is determined also by generating function F of the Kerr theorem, (20),

$$Z^{-1} = \tilde{r} = -\partial_Y F = v + ia. \quad (26)$$

One sees that expansion tends to zero only near the front plane $v = 0$, (or $z = t$), where the twist ω is maximal. In the vicinity of the axis z , where $Y\bar{Y} \rightarrow 0$, and far from the front plane, congruence tends to simple form $e^3 = du$ corresponding to pp-wave solutions.

For the twist-free case $a = 0$, congruence represents gradient of scalar function $e^3 = d(u + \zeta\bar{\zeta}/v)$, and on the

front surface, $v = 0$, it is singular, showing that the front $z = t$ is a shock-plane. However, it should be emphasized that for the spinning solutions, $a \neq 0$, front is smooth.

If function Z is determined, electromagnetic field (13) is defined by two functions A and γ which satisfy to equation (14). It is easy to check that function obeys the condition $Y_{,3} = 0$. It yields simplification of the operator (16) which reduces to $\mathcal{D} = \partial_3$. As a result, the equations (14), and (15) take the simple form

$$A_{,2} = A_{,4} = 0, \quad (27)$$

$$A_{,3} + \bar{Z}^{-1}\gamma_{,2} = 0, \quad \gamma_{,4} = 0. \quad (28)$$

As it follows from (13), the function γ describes a null electromagnetic radiation propagating along the Kerr congruence (direction e^3). All the considered in [13] and obtained up to now twisting Kerr-Schild solutions were restricted by the case the $\gamma = 0$, which corresponds to the stationary and radiation-free electromagnetic fields. We will also presume here that $\gamma = 0$ and will discuss problem of the solutions with $\gamma \neq 0$ in the next section. Since we have $Y_{,2} = Y_{,3} = Y_{,4} = 0$, the equations (27), and (28) reduce now to

$$A_{,2} = A_{,3} = A_{,4} = 0, \quad (29)$$

which means that A may only be an arbitrary holomorphic function of the complex function $Y(x)$.

The gravitational equations (17), and (18) are also simplified taking the form

$$M_{,2} = 0, \quad (30)$$

$$M_{,3} = M_{,4} = 0 \quad (31)$$

with extra condition that M is real. The unique corresponding solution is $M = \text{const.} = m$. Therefore, in spite of the very nontrivial form of the congruence, but due to extra property $Y_{,3} = 0$, the equations turn out to be very simple and easily solvable. As a result, metric is determined by the Kerr-Schild ansatz (5) with function h given by

$$h = [mv - \frac{1}{2}A(Y)\bar{A}(\bar{Y})]/(v^2 + a^2), \quad (32)$$

where $Y = \frac{x+iy}{2^{1/2}(v+ia)}$, and

$$e^3 = du - \frac{x^2 + y^2}{2(v^2 + a^2)}[dv + 2v(xdx + ydy) + 2a(ydx - xdy)]. \quad (33)$$

Summarizing, we can represent metric in the form

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2 + 2\frac{mv - A(Y)\bar{A}(\bar{Y})}{v^2 + a^2} \times \{du - \frac{x^2 + y^2}{2(v^2 + a^2)}[dv + 2v(xdx + ydy) + 2a(ydx - xdy)]\}^2 \quad (34)$$

Electromagnetic field is given by $F_{\mu\nu} = \Re \mathcal{F}_{\mu\nu}$, where $\mathcal{F}_{\mu\nu} = \mathcal{F}_{ab}e^a e^b$, is the self-dual field having the following nonzero tetrad components

$$\mathcal{F}_{12} = \mathcal{F}_{34} = A/[v + ia]^2; \quad \mathcal{F}_{31} = -A'_Y/[v^2 + a^2], \quad (35)$$

and

$$\mathcal{F}_{23} = \mathcal{F}_{14} = \mathcal{F}_{24} = 0. \quad (36)$$

The solutions with with $A = Y^n$, $n = -1, -2, \dots$ are singular by $Y \rightarrow 0$. Similar to 'gyrons', they represent the spinning light-like beams with singular strings positioned along the z-axis. Near this string, $Y \rightarrow 0$, the Kerr congruence takes the constant direction $e^3 = du$, and solution tends to the well known A. Peres pp-wave solutions [4, 15].

Note, that the ordinary Kerr geometry has a quadratic in Y Kerr function $F(Y)$. As a consequence there are two roots of the solution $F = 0$, two different principal null directions corresponding to type D of metric and the known twofoldedness of space with a branch line on the Kerr singular ring. This structure is retained by a boost, however, in the limiting case $v = c$ light-like solution F is linear in Y . Therefore, the degree of function F changes by jump when the coefficient p becomes equal to zero in the ultrarelativistic limit. This fact shed a light on the difficulties with the obtaining of the ultrarelativistic limits, which are related with *non-smoothness* of the limiting procedure [9]. In the limit $v = c$, the Kerr twofoldedness disappears by jump and space-time unfolds. The position of the "negative" sheet of the Kerr solution corresponds to the negative values of the radial distance. In the considered example $Re \tilde{r} = 2^{-1/2}(z - t)$, which shows that the "positive" and "negative" Kerr's sheets are placed on the different half-spaces divided by the front-surface $z = t$. The "negative" sheet becomes the sheet of advanced fields placing before the front of solution.

It should be noted that one can alternatively consider solutions with Killing vector $\hat{K} = \partial_v$, which correspond to the front $u = const.$ moving in opposite directions $z = const. - t$. Corresponding parameters

$$c = q = \bar{q} = 0, \quad p = 1, \quad (37)$$

lead to quadratic generating function

$$F = -iaY - Y(u + Y\bar{\zeta}), \quad (38)$$

and to function

$$P = Y\bar{Y}. \quad (39)$$

However, analysis shows that this case does not differ from the linear one considered before[29].

The equation $F(Y) = 0$ will have two solutions, and one of them, $Y = 0$, is nonphysical one, since it leads to $P = 0$ and to infinite value for the tangent vector to Kerr congruence $k_\mu = e_\mu^3/P$.

Assuming that $\bar{Y} \neq 0$, we obtain another solution

$$Y = -(ia + u)/\bar{\zeta} \quad (40)$$

which is related to previous one by the space reflection $P : (x, y, z) \rightarrow -(x, y, z)$ and antipodal inversion $J : Y \rightarrow -1/\bar{Y}$. The 'orientifold' transformation $\Omega = P \cdot J$ plays important role in the particle physics and in superstring theory (see for example [16]). It appears in the complex Kerr geometry as a necessary element of the complex Kerr string [17, 18, 19].

Finally, one can easily calculate the five complex quantities determining the degeneracy of the conformal curvature tensor. By construction, the Kerr-Schild metrics are algebraically special and $C^{(4)} = C^{(5)} = 0$.

We have also

$$\begin{aligned} C^{(1)} &= -h_{,11}, \\ C^{(2)} &= 2Zh_{,1}, \\ C^{(3)} &= 2Z[h_{,4} + (\bar{Z} - Z)h]. \end{aligned}$$

Since $Y_{,3} = 0$, we obtain $Z_{,2} = Z_{,1} = 0$, and in the particular case $A = const.$ we obtain also $h_{,1} = h_{,11} = 0$, and consequently, $C^{(1)} = C^{(2)} = 0$, and $C^{(3)} = -mZ^3 + e^2Z^3\bar{Z} \neq 0$.

IV. DISCUSSION

We described the twisting light-like solutions which are direct descendants of the Kerr-Newman solution and obtained by a smooth transfer in the parameters of the generating function of the Kerr theorem. Contrary to corresponding analogs of the Aichelburg-Sexl solutions, these solutions are twisting, have a non-zero angular momentum and a smooth (non-singular) front surface.

The parameters m and a of these solutions correspond to their relativistic limiting values, i.e. $m = E$ and $a = J/m$, obtained by the condition of the finite value of angular momentum $J = ma$. During the light-like limit $v \rightarrow c$, parameter a_0 , radius of the Kerr singular ring, turns out to be shifted to infinity, and its role is going to the finite relativistic parameter $a = a_0\sqrt{1 - (v/c)^2}$ which acquires principal geometrical meaning, parametrizing the generating function F of the Kerr theorem and defining the twist of the null congruence. Sources of the light-like solutions turn out to be shifted into complex direction $z \rightarrow z + ia$, just like the complex source of the Kerr solution [9, 19, 21].

In the light-like case, the Kerr relation $J = ma$ takes the form $E = J/a$, and if we set $J = \hbar$, assuming that this solution describes gravitational field of a photon, one obtains that a is de Broglie wavelength of the corresponding photon relation $E = \hbar\nu$. Together with the obtained by

Carter double gyromagnetic ratio of the Kerr-Newman solution and with the obtained therein stringy structures [17, 18, 19], as well as with the reach spinor-twistor structure connected to the Dirac equation [18, 20, 21], it gives one more evidence to the relationships between the Kerr geometry and structure of spinning particles.

The considered here case of linear in Y generating function F is important, since it is simpler of the quadratic one, which allows one to expect a progress in the analysis of the case $\gamma \neq 0$ related with the wave electromagnetic fields [21].

Contrary to the Kerr case, the periodic stringlike structure, which could provide a resonance frequency for electromagnetic field, is absent in this case. One can expect, that such structure could appear in some multidimensional generalizations of the presented solutions. It seems that such a generalization may be obtained by unification of the Kerr-Schild and Brinkman metrics.

Interest to these solutions is supported for a few reasons related with the twistorial approach to the problem of gravitational (and electromagnetic) interactions of spinning particles. The corresponding Kerr-Schild treatment of multi-particle solutions [12] is based on the Kerr theorem with generating functions F of different degrees in Y , including the linear ones. Twistorial structure of a Kerr-Newman particle is described by a quadratic in Y generating function $F = F_2(Y)$, (21), and this case was investigated in details. It was shown [9, 13, 14] that quadratic in Y functions correspond to isolated spinning particles with arbitrary position, orientation of spin and (finite) boost. Contrary, the solutions with generating functions of first degree in Y , $F = F_1(Y)$ have not paid considerable attention before, in spite of their important physical significance - relations to the light-like spinning particles. The general case of higher degrees in Y , considered in [12], showed that multi-particle Kerr-Schild solutions lead to a multi-sheeted twistor space which may be split into simple one- and two-sheeted blocks corresponding to a set of the light-like and massive particles. In particular, for the generating functions of third degree, $F_3(Y)$, space-time will be three-fold and may be considered as a product of the linear in Y function $F_1(Y)$, given by (23), and quadratic in Y , function $F_2(Y)$, given by (21),

$$F_3(Y) = F_1(Y) \cdot F_2(Y). \quad (41)$$

Then the equation $F_3 = 0$ will determine two independent twistorial structures belonging to different Riemannian sheets of the function $Y(x)$. One of them corresponds to solution $Y_1(x^\mu)$, $x^\mu \in M^4$, of the equation $F_1(Y) = 0$, and second one - to a twofold solution $Y_2(x^\mu)$ of the equation $F_2(Y) = 0$. Although these structures

are independent, machinery of the Kerr-Schild formalism shows that the gravitational and electromagnetic fields of the particles 1 and 2 interact, forming a singularity at their *common* twistor line, fixed by the set [12]

$$\{x^\mu : Y_1(x^\mu) = Y_2(x^\mu)\}. \quad (42)$$

In other words, solution acquires a pole (propagator)

$$\sim \frac{1}{Y_1(x^\mu) - Y_2(x^\mu)}, \quad (43)$$

which is exhibited in the form of a singular null-string beam between the particles 1 and 2.

This consideration is close related to the suggested by Nair [25] and renewed by Witten [24] twistor-string approach to perturbative gauge theory of quantum scattering, where the traditional quantum treatment in momentum space gets a natural generalization to twistor space with corresponding twistorial generalizations of the wave functions and amplitudes of scattering. The gauge bosons are described by curves of first degree in twistor space, and the corresponding plane waves are replaced by twistor null planes.

The considered here approach based of Kerr theorem seems to be more informative, since description of the wave function by a single twistor null plane, is replaced by section of a twistor bundle. We expect that the twistorial theory of scattering based on the Kerr theorem acquires essential advantages and may represent a new background for quantum theory of the gauge and massive spinning particles.

Besides, since twistorial description of the wave functions contains important coordinate information, it represents a very natural way to incorporate gravity in quantum theory [20].

Finally, the light-like beam solutions demonstrate an universality, appearing in many different areas. In particular, they appear as beams or pp-waves in gravity [5, 11, 15, 21], as pp-strings, Schild strings or chiral constituents of twistor-string in string theory [18, 24, 25], as singular twistor lines by interaction of spinning particles [12], as well as by excitations of the rotating black-holes [26]. They possess many remarkable classical and quantum properties which allow one to suppose their fundamental role in the processes of interaction and, apparently, in the structure of vacuum.

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[27] One can naively suppose that the limit $m_0 \rightarrow 0$ becomes flat. Indeed, it differs from flat topologically. In particular, in the Kerr case this limit may be performed with a constant parameter a , leading to a space-time with twofold topology having the branch line along the naked Kerr singular ring. The corresponding Schwarzschild solution may be obtained by the subsequent limit $a \rightarrow 0$, leading to two copies of Minkowski space glued at the singular point (line) $x = y = z = 0$. Alternatively, changing the order of limits (doing $a = 0$ first), one obtains one exemplar of the space with a punctured point.

[28] In explicit form they are $\partial_1 = \partial_\zeta - \bar{Y}\partial_u$; $\partial_2 = \partial_{\bar{\zeta}} - Y\partial_u$; $\partial_3 = \partial_u - h\partial_4$, and $\partial_4 = \partial_v + Y\partial_\zeta + \bar{Y}\partial_{\bar{\zeta}} - YY\partial_u$.

[29] There are also the simplest cases $F = Y + c$, which correspond to pp-wave solutions.